



PRI Handbook:
A Guide on Mathematical
Exhibits



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Introduction

Mathematics is a key part of STEAM subjects and one of the main skills needed today and in the future to awaken scientific vocations among young people. The acronym SMEM used for this project stands for “Significant Mathematics for Early Mathematicians”. The SMEM project has taken on a multidimensional approach that aims to create a new space for innovative teaching methods in mathematics, reduce the gender gap related to STEM-oriented pathways, cultivate a variety of soft and human-centric skills and foster a positive image of mathematics as a subject. The exhibition is addressed to children aged between 3 and 8 years, as well as their teachers, but also to anyone who is interested in bridging the gap between mathematics and play.

The project is proposed from the point of view of non-formal education, which we could summarise as: “We don't teach, but they learn”. As such, this creates a virtuous circle of the “Hands-on, Minds-on, Hearts-on and Talk-on” experience.

In this sense:

- the proposed activities are undirected;
- the information is based on suggestions rather than instructions;
- the tasks are not explicit, leaving a large space for users to interact with the material.

In this regard, it should be mentioned that the nature of the exhibits is based on inclusive design as described by CAST’s (2018) conceptualisation of [Universal Design for Learning Version 2.2](#), where there are multiple means of presentation, engagement and expression.

Specific tasks, especially if they present greater difficulties, can be introduced by the facilitator in the exhibition time, such as spontaneous workshops or on special occasions such as teacher training. The rationale behind the exhibits is to allow users to engage with mathematical concepts and skills through seemingly simple challenges. Moreover, through this project, we are able to verify the effectiveness of hands-on physical and virtual exhibits and evaluate the results based on engagement, competence and skill development, as well as promoting a deeper understanding of the concepts presented. These aspects are a work in progress that is open to the contribution of the educational community.

Drawing Dice

Age Group

3-8



Material

Four wooden dice. They should have different shapes marked in one contrasting colour on each side. All the dice are equal. The same goes for the order of drawings on all sides.

Alternatively, the dice could be of origami paper or cardboard and glue, with the shapes painted on the sides. The choice of shapes drawn on each side could change to enable the creation of new and different figures.



Activity

The activity is to place the four dice in such a way as to get a figure with dimensions $2 \times 2 \times 1$. The recognizable shape should appear on the top when you join their upper four sides.

We have already proposed some shapes to be formed (square, triangle, parallelogram, star, etc.), but during the play, you could discover a lot of new figures.

Further Explorations

One of the activities that arise immediately: try to calculate the perimeter and the area of the shapes created. Since all the forms drawn on each side of the dice are “touching” the middle point or the vertex, it’s easy to figure out the perimeter and the area without using formulas, just reasoning.

Another challenge you could try: position the dice with your new, original graphics swapped with the proposed shapes on any of the sides (by using origami or cardboard and glue when building the cubes).

This one is a bit advanced, but you could try to count the number of different figures we can create with these dice.

Mathematical Background

Geometry, exploration of basic shapes.

Spatial view, experimenting with rotation, creation of new figures.

Symmetrical and non-symmetrical shapes.

Some combinatorial concepts.

Transferable Skills

Stimulating creativity, maths research, and new questions.

Fruit Harvesting

Age Group

3-8



Material

Two circular boards, with ordained series of holes to hold the “cherries.” The boards are integral parts of two baskets. Fix them at the bottom of both baskets.

For cherries, we recommend using wooden balls, single or in sets of 2, 3, and 4 (connected with wooden pegs), with the following distribution:

Single	Double	Set of 3	Set of 4	Total
3	2	2	2	21

Activity

The double exhibit explicitly enables one of the boards to be always empty, so a new player can always have a basket to fill with the groups of cherries.

The same thing also happens for the solution for each of the baskets, so even if a player has already cracked the challenge, there is an option to search for a new result.

Furthermore, you could create a kind of instant workshop where challenges can be posed, such as: “You cannot use the group of three cherries to fill the group of three holes” or similar.

With the oldest pupils, you could try the collaborative task of finding all the possible distributions of cherries.

Further Explorations

While acquiring the concept of numbering and calculation skills, it is especially significant to practise activities that allow the transition from the number as a sum of units to its perception as a continuous and compact quantity, in other words, from counting to calculation.

In this sense, it is valuable to work in class by constructing numbers as a sum of units (using, for example, multilink cubes) and as "compact" entities (using the Cuisenaire rods). This model should always remain available and serviceable to respond to doubts.

In terms of the combination of numbers, you could reinforce the concepts by using a simple abacus and then moving on to the stimulus of mental calculation.

Mathematical Background

Numeracy, the strategy of counting numbers.

Introduction to the strategy of addition.

Mental calculation.

Basics combinatorial concepts.

The Beaver's Dam

Age Group

3-8



Material

You will need eight wooden rectangular cuboids and one wooden cube. Cuboids should be three times longer than cubes (so, dimensions of $3a \times a \times a$), where a is the side of the cube. You will also need a square template (grid) on the board to use it in the background.

Activity

The task is to place the wooden elements (all the cuboids and the cube) in such a way that they form a shape with a square as a base. The children can use the grid in the background as guidance. It suggests the size and shape of the flank of the finished structure.

There is one unique solution visible in the picture above. The small cube has to be in the middle of the square.

Further Explorations

A further activity would be to count how many cubes match one of the cuboids. Then the children could count how many cubes one would need to replace all the cuboids and fill out the dam.

Another challenge could be to figure out whether it is possible to fill a square of size $3a$ or one of size $4a$ by using the cube and some of the cuboids.

This one is a bit advanced, but could you imagine covering a square of size $7a$ with a cube of size a and a required number of cuboids with dimensions of $4a \times a \times a$?

Mathematical Background

Geometry, the exploration of basic forms.

Spatial view, experimentation with rotations.

Pattern recognition.

Manifolds, number decomposition.

Learning the mathematical language.

Transferable Skills

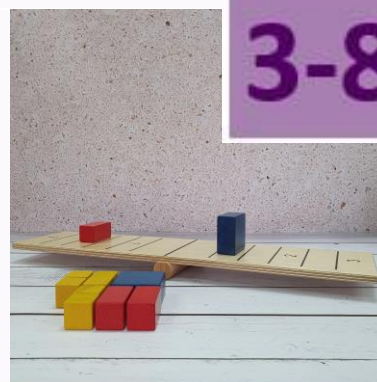
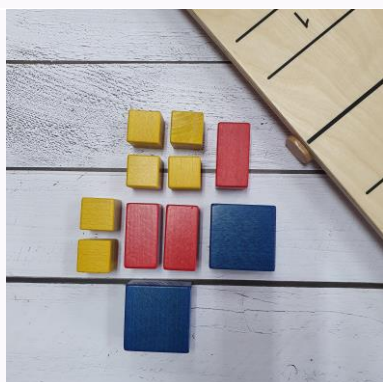
Stimulating creativity, maths research, and new questions.

Improving problem-solving skills.

The Seesaw

Age Group

3-8



Material

This exhibit consists of a wooden dandle board (board balanced on centre support) with numbers from one to three on each side. Additionally, it includes 11 wooden bricks with three different sizes (and, therefore, different weights). Bricks of the same size are of the same colour.

Activity

The first activity will rise spontaneously: the children will immediately place bricks somewhere on the lever arm while looking for equilibrium. You could suggest using only a few bricks (for instance, 3 or 4), so they could intuitively adopt the underlying rules.

The second activity could be to put bricks of diverse colours on opposite sides of the seesaw and achieve equilibrium. Of crucial importance is to lay different bricks on the same number on each side (say, on the '2'). The children can then discover that larger-sized bricks are heavier than smaller ones.

The third challenge: place the cuboids on different numbers on the lever arms and establish balance. If we consider the smallest brick as a unit and the two larger-sized bricks as two and four units, we could get the equilibrium by using the following rule:

The product of the number of units and the number on which you place the bricks must be the same on both sides of the lever arm (for instance, put the brick worth four units on number 1 and one medium-sized brick representing two units on number 2).

Further Explorations

A convenient follow-up activity would be to weigh different objects that kids will look for in the classroom. They should be small enough to fit on the scale.

Another interesting observation is to compare the exhibit to a seesaw the children can find on a playground. They could notice that two children seem to weigh less than one if one kid is sitting on the very end of the lever arm and the two children are closer to the middle.

The children could also practise simple multiplication and division of numbers (see the reasoning in the Activities section).

Mathematical Background

Decompositioning numbers, practising multiplication tables for the first few numbers.
Learning the mathematical language.

Transferable Skills

Stimulating creativity, maths research, and new questions.
Practising fine motor skills.

Looking For An Equilibrium



Age Group

4-8

Material

You will need one wooden plate fixed vertically on a stand. In addition, we suggest six different geometrical shapes made out of PVC (or cardboard), which can balance on the plate.

Activity

Balance the figures on the structure one by one. There is no unique solution for any of the shapes from the set: you could stabilise them differently. Nevertheless, they all have one thing in common: the centre of gravity is a point that always lies on the wall.

Further Explorations

Following up on the exhibit, several activities for the classroom are available.

The children could search for objects in the classroom they could balance or do a scavenger hunt for things with similar shapes in their environment.

Another challenge could be to copy the objects on paper, cut the shapes, and then place them on the plastic figures. They should again equipoise the structures and draw a line on the paper piece along which the object hangs on the edge. They could also examine the figures for symmetry.

The last exercise could be to balance a rod with two hands. You could easily find the centre of gravity if you lay a rod on two fingers, one of each hand, and then move both hands toward each other. Surprisingly, the rod keeps in balance. By joining the fingers together, you have found the centre of gravity.

Mathematical Background

Geometry, the exploration of basic forms, forms recognition.

Symmetric shapes with respect to a line and with respect to a point.

Centre of gravity.

Learning the mathematical language.

Transferable Skills

Stimulating creativity, maths research, and new questions.

Practising hand-eye-coordination.

Representing Numbers

Age Group

3-8



Material

A wooden board that has ten holes for placing tokens. Mark the holes with dots. The number of dots corresponds to the numbers from 1 to 10.

Ten tokens with different images represent numbers from 1 to 10.

Activity

The child should take token by token and place them in the hole corresponding to the number represented by the token picture.

Therefore, this activity allows teachers to work with different mathematical notions: number sense and numeration by learning to count up to an arbitrary number and recognising numbers.

This activity will allow the child to associate a picture with a number of objects identical to a number on the board. There is no wrong answer as long as it engages in a dialogue.

Further Explorations

The children could play in pairs: one could distribute the tokens, the other could perform the cross checking and vice versa.

Another option could be to do a scavenger hunt in pairs, one kid sets the target number, the other one searches for the object, and they change the roles for every number.

Mathematical Background

Learning the mathematical language.

Number identification.

Comparing the objects and learning the numbers from 1 to 10.

Transferable Skills

Stimulating creativity, maths research, and new questions.

Creating umbrellas

Age Group

3-8



Material

Hybrid exhibit (physical and virtual). On the tablet, an app allows to draw simple shapes, and the barycenter is displayed. On the physical table, some transparent pieces in the shapes of leaves, a vertical stick, and optionally a non-permanent marker pen.

Activity

The goal is to balance a leaf horizontally on the top of the stick, creating an “umbrella”:

1. Place a transparent leaf shape on top of the tablet screen.
2. Follow the contour of the shape, drawing it on the app. The barycenter is seen at all stages.
3. When the drawn shape is closed, it changes color and freezes. Click on the pencil to restart.
4. With the help of the marker, mark the barycenter point on the plastic piece (or just hold a finger on top of the point).
5. Transfer the shape from the top of the tablet to the stick, positioning it over the barycenter.

Further Explorations

The educator may use this app to spark reflection about the existence of such a “magical point”, the only point allowing to hold a shape balanced in equilibrium. The educator can also point out several connections between this exhibit and others in the exhibition (Looking for an equilibrium, seesaw). See the Educator’s Guide for more information.

Mathematical topics

Barycenter, averages (arithmetic mean, weighted mean), lever principle, geometric shapes.

Transferable skills

Exploring mathematical properties, following a procedure to find a solution, making conjectures to explain a phenomenon, practising fine motor skills.

Heart in the sky

Age Group

3-8



Material

Virtual exhibit. The app shows a kite in the sky. An icon of a cloud with a blowing face represents the wind. Click and drag close to the blowing cloud to make wind, and the kite will fly in the direction of the wind (the direction of the kite and the direction of the wind blowing from the cloud are parallel). The intensity of the wind is represented by a purple fog coming out of the cloud. The stronger the wind, the faster the kite flies.

Activity

Control the wind to make the kite fly following the outline of the shape. When you succeed, Amy the fox will appear to greet you, and a new shape will be proposed.

The child needs to interiorize the idea of direction in order to be able to control the kite. Help the child by using short touches to see the effect of the wind, release the finger to stop the wind and think. Straight sides are easier. Use corners to explain changes in direction. Curves are trickier, move slowly.

Further Explorations

As an educator, you can explain here the physics concepts of direction (unit vector), speed (scalar), and velocity (vector). Next, you can trigger a conversation relating direction and trajectory. You can compare the app with the steering wheel in a car, does it work similarly?

For early-age children, the goal is not to give precise physical definitions, but to distinguish different phenomena, and to realize that through some physical/mathematical connection, they relate to each other. See the Educator's Guide for a more detailed discussion.

Mathematical topics

Distance action, encoding information, direction, speed, velocity, accumulation effect (integration).

Transferable skills

Coordination eye-hand, fine motor skills.



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